

2002 - Higher Paper I

1) Centre = $(-1, 1)$

$$M_{\text{radius}} = \frac{1-3}{-1-2} = \frac{-2}{-3} = \frac{2}{3}$$

$$M_r \times M_t = -1 \quad \therefore M_t = \underline{\underline{\frac{-3}{2}}}$$

$$y-b = m(x-a) \quad m = \frac{-3}{2}, (a, b) = (2, 3)$$

$$\underline{\underline{y-3 = \frac{-3}{2}(x-2)}}$$

2)

	P	Q	R	
x:	-1 → -4	3	2 ← 5	$\frac{6}{3} = \underline{\underline{2}}$
y:	-1 → 2	1	1 ← 2	$\frac{3}{3} = 1$
z:	0 → -2	-2	1 ← -3	$\frac{3}{3} = 1$

$$\therefore \underline{\underline{Q(3, 1, -2)}}$$

3) a) i) $f(g(x)) = f(\sin x) = \underline{\underline{\sin 2x}}$

ii) $g(f(x)) = g(\sin x) = \underline{\underline{2 \sin x}}$

b) $2 \sin 2x = 2 \sin x$

$$2 \sin 2x - 2 \sin x = 0$$

$$4 \sin x \cos x - 2 \sin x = 0$$

$$2 \sin x (2 \cos x - 1) = 0$$

$$2 \sin x = 0 \quad \cos x = \frac{1}{2}$$

$$x = 0, 180, 360 \quad x = 60, 300$$

$$\underline{\underline{x = 0, 60, 180, 300, 360}}$$

4) $M = \tan 45^\circ = 1.$

$M = \frac{dy}{dx} = 4x - 7 = 1$

$\therefore x = \frac{8}{4} = \underline{\underline{2}}$

$y = 2x^2 - 7x + 10$

$= 8 - 14 + 10 = 4$

$\therefore \underline{\underline{(2, 4)}}$

5) $AC = \sqrt{2} \quad CB = \sqrt{10}$

$\sin a = \frac{1}{\sqrt{2}}, \cos a = \frac{1}{\sqrt{2}}, \sin b = \frac{1}{\sqrt{10}}, \cos b = \frac{3}{\sqrt{10}}$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

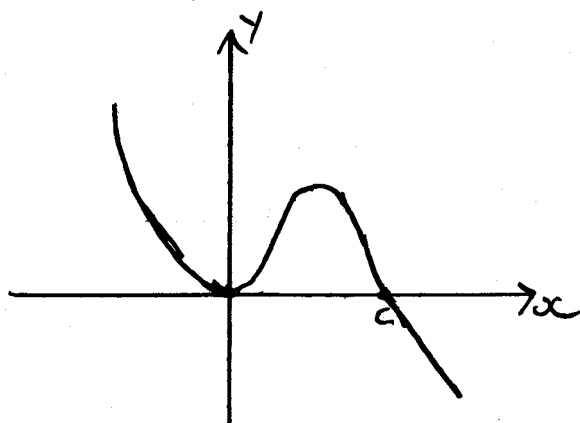
$= \frac{1}{\sqrt{2}} \times \frac{3}{\sqrt{10}} + \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{10}}$

$= \frac{1}{\sqrt{2}} \left(\frac{3}{\sqrt{10}} + \frac{1}{\sqrt{10}} \right)$

$= \frac{1}{\sqrt{2}} \times \frac{4}{\sqrt{10}} = \frac{4}{\sqrt{2}\sqrt{10}} = \frac{4}{\sqrt{20}}$

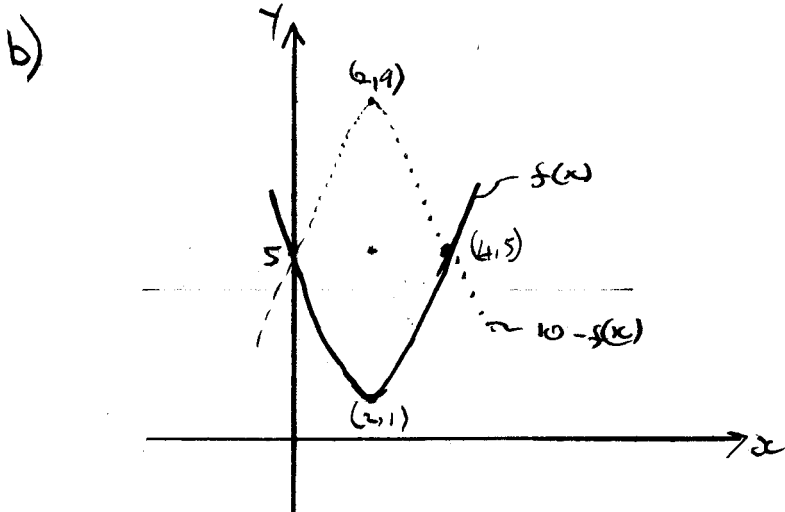
$= \frac{4}{\sqrt{4}\sqrt{5}} = \frac{4}{2\sqrt{5}} = \underline{\underline{\frac{2}{\sqrt{5}}}}$

6)



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$$\begin{aligned} 7) \text{ a) } f(x) &= x^2 - 4x + 5 = (x^2 - 4x + 4) + 5 - 4 \\ &= \underline{(x-2)^2 + 1} \end{aligned}$$



$$\begin{aligned} \text{c) } 10 - (x-2)^2 - 1 &= 0 \\ (x-2)^2 &= 9 \\ x-2 &= \pm 3 \\ x &= -1, 11 \\ \therefore \underline{\underline{-7 \leq x \leq 11}} \end{aligned}$$

8) a) $y = 2 \cos 2x$

b) $2 \cos 2x = -\sqrt{3}$

$$\cos 2x = \frac{-\sqrt{3}}{2}$$

$$R.A = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$$

$$\therefore 2x = \frac{5\pi}{6} \quad \text{OR} \quad \frac{7\pi}{6}$$

$$x = \frac{5\pi}{12} \quad \text{OR} \quad \frac{7\pi}{12}$$

$\therefore 2$

$$\therefore \underline{\underline{B\left(\frac{7\pi}{12}, -\sqrt{3}\right)}}$$



$$\begin{aligned}
 \text{a) a) } \sin x - \cos x &= k \sin(x-a) \\
 &= k(\sin x \cos a - \cos x \sin a) \\
 &= k \sin x \cos a - k \cos x \sin a \\
 \therefore k \cos a &= 1 \quad k \sin a = -1
 \end{aligned}$$

Square and add: $k^2(\cos^2 a + \sin^2 a) = 2$

$$\underline{k = \sqrt{2}}$$

$$\frac{k \sin a}{k \cos a} = \frac{-1}{1} = \underline{\underline{\tan a = -1}}$$

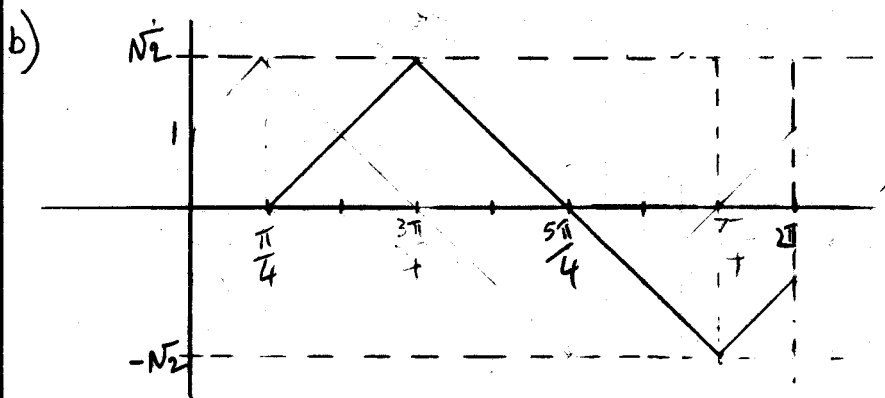
$\therefore \tan a = -1$ \sin is +ve, \cos is +ve.
 \therefore 1st Quadrant

$$\therefore a = \tan^{-1}(-1) = \frac{\pi}{4}$$

$$\therefore a = 2\pi - \frac{\pi}{4}$$

$$\therefore \sin x - \cos x = \underline{\underline{\sqrt{2} \sin(x - \frac{\pi}{4})}}$$

$$\therefore \underline{\underline{k = \sqrt{2} \quad a = \frac{\pi}{4}}}$$



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10) a) $f(x) = (8 - x^3)^{1/2}$

$$f'(x) = \frac{1}{2}(8 - x^3)^{-1/2} \times -3x^2$$

$$= \frac{-3x^2}{2\sqrt{8 - x^3}}$$

b) $\int \frac{x^2}{(8 - x^3)^{1/2}} = \underline{\underline{\frac{-2}{3}(8 - x^3)^{1/2} + C}}$

11) If $y = k \cdot 5^n$ then $n = M$, $C = \log_5 k$

$$M = \frac{-1}{0.5} = \underline{\underline{-2}}$$

$$C = \log_5 k$$

$$1 = \log_5 k$$

$$\therefore \underline{\underline{n = -2}}$$

$$\log_5 5 = \log_5 k$$

$$\therefore \underline{\underline{k = 5}}$$

$$\therefore \underline{\underline{y = 5x^{-2}}}$$

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1) a) Mid Point AB = $(-2, 2)$

$$M_p = \frac{2-2}{5-2} = \frac{0}{3} = 0$$

\therefore P.C.S $y=2$

b) $M_{bc} = \frac{2-2}{5-3} = \frac{4}{2} = 2 \quad \therefore M_1 M_2 = -1 \quad \therefore M_3 = -2$

Mid Point BC = $(1, 0)$

$$y-b = m(x-a) \quad m = -2, \quad (a, b) = (1, 0)$$

$$y-0 = -2(x-1)$$

$$\underline{y = -2x + 2} \quad \therefore \underline{y = 2 - 2x}$$

c) $y=2 \quad \therefore \quad 2 = 2 - 2x$
 $4 = 2 - 1$
 $3 = 2 - 1$
 $x=0$

\therefore $(0, 2)$

2) a) B(6, 6, 0)

b) $\vec{DA} = a-d = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ 3 \\ 8 \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \\ -8 \end{pmatrix} \quad \vec{DB} = b-d = \begin{pmatrix} 6 \\ 6 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ 3 \\ 8 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ -8 \end{pmatrix}$

c) $\cos \phi = \frac{a \cdot b}{|a||b|} = \frac{\vec{DA} \cdot \vec{DB}}{|\vec{DA}||\vec{DB}|}$

$$\vec{DA} \cdot \vec{DB} = (3 \times 3) + (-3 \times 3) + (-8 \times -8) = 9 - 9 + 64 = \underline{64}$$

$$|\vec{DA}| = \sqrt{3^2 + (-3)^2 + (-8)^2} = \underline{9.055} \quad |\vec{DB}| = \sqrt{3^2 + 3^2 + (-8)^2} = \underline{9.055}$$

$$\therefore \cos \phi = \frac{64}{9.055 \times 9.055} = 0.781$$

$$\therefore \phi = \cos^{-1}(0.781) = \underline{38.6^\circ}$$

3) a) Max at $\frac{dy}{dx} = 0$: $\frac{dy}{dx} = 6x^2 - 14x + 4 = 0$
 $2(3x^2 - 7x + 2) = 0$
 $2(3x - 1)(x - 2) = 0$
 $\therefore x = \frac{1}{3}$ OR $x = 2$

\therefore at Max $x = \frac{1}{3}$

b)

2	2	-7	4	4
		4	-6	-4
	2	-3	-2	0

$\therefore (x-2)(2x^2-3x-2)$
 $(x-2)(2x+1)(x-2)$

c) $A_x = -\frac{1}{2}$, $A_y = 0$
 $\therefore A(-\frac{1}{2}, 0)$
 \therefore $x < -\frac{1}{2}$

4) a) $u_{n+1} = au_n + b$
 $u_{n+1} = 0.8u_n + 0.5$ $-1 < 0.8 < 1 \therefore$ limit exists.
 Limit $L = \frac{b}{1-a}$
 $L = \frac{0.5}{1-0.8} = \frac{0.5}{0.2} = \underline{2.5m}$

b) $L = 2$ find a
 $\therefore 2 = \frac{0.5}{(1-a)} = 2(1-a) = 0.5$
 $2 - 2a = 0.5$
 $-2a = -1.5$
 $a = 0.75$

\therefore Trim 25%

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5) Find limits: $1+10x-2x^2 = 1+5x-x^2$

$$x^2 - 5x = 0$$

$$x(x-5) = 0$$

$$\underline{x=0}, \underline{x=5}$$

$$\begin{aligned}\therefore \text{Area} &= \int_0^5 (1+10x-2x^2) - (1+5x-x^2) dx \\ &= \int_0^5 (5x-x^2) dx = \left[\frac{5x^2}{2} - \frac{x^3}{3} \right]_0^5 \\ &= \left(\frac{5 \times 5^2}{2} - \frac{5^3}{3} \right) - 0 = 62.5 - 41.7 \\ &= \underline{\underline{20.8 \text{ units}^2}}\end{aligned}$$

6) $M = \frac{dy}{dx} = 2 \cos\left(x - \frac{\pi}{6}\right)$ at $\frac{\pi}{3}$

$$\begin{aligned}M &= 2 \cos\left(\frac{\pi}{3} - \frac{\pi}{6}\right) = 2 \cos\left(\frac{\pi}{6}\right) \\ &= \frac{2\sqrt{3}}{2} = \underline{\underline{\sqrt{3}}}\end{aligned}$$

at $x = \frac{\pi}{3}$: $y = 2 \sin\left(\frac{\pi}{6}\right) = 2 \times \frac{1}{2} = \underline{\underline{1}}$

$$\begin{aligned}y-b &= m(x-a) \quad m = \sqrt{3} \quad (a,b) = \left(\frac{\pi}{3}, 1\right) \\ \underline{\underline{y-1}} &= \underline{\underline{\sqrt{3}\left(x - \frac{\pi}{3}\right)}}$$

7) Cuts x-axis at $y=0$

$$\therefore \log_3(x-2) + 1 = 0$$

$$\log_3(x-2) = -1$$

$$\log_3(x-2) = \log_3 \frac{1}{3}$$

$$x-2 = \frac{1}{3}$$

$$\underline{\underline{x = 2\frac{1}{3}}}$$

$$-1 = \log_3 \frac{1}{3}$$

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8) If $a = \frac{dv}{dt}$ then $v = \int a dt$

$$v = \int 2(4-t)^{1/2} dt = \frac{2}{-1 \times \frac{3}{2}} (4-t)^{3/2} + C$$

$$= -\frac{4}{3} (4-t)^{3/2} + C$$

$$= \frac{-4\sqrt{(4-t)^3}}{3} + C$$

at $t=0, v=0$

$$0 = \frac{-4\sqrt{(4)^3}}{3} + C$$

$$0 = -10.67 + C$$

$$\therefore C = \underline{\underline{10.67}}$$

$$\therefore v = \underline{\underline{\frac{-4\sqrt{(4-t)^3}}{3} + 10.67}}$$

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9)

$$b^2 - 4ac \geq 0$$

$$a = (1-2k) \quad b = -5k \quad c = -2k$$

$$= (-5k)^2 - (4 \times (1-2k) \times -2k)$$

$$= 25k^2 - (-8k(1-2k))$$

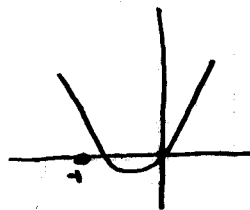
$$= 25k^2 - (-8k + 16k^2)$$

$$= 25k^2 + 8k - 16k^2$$

$$= \underline{9k^2 + 8k} \geq 0$$

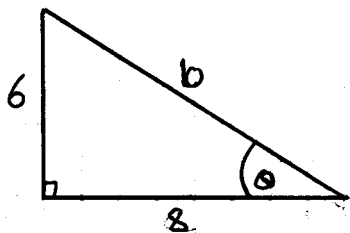
$$\therefore k(9k+8) \geq 0$$

$$k = 0 \quad \text{or} \quad k = -\frac{8}{9}$$

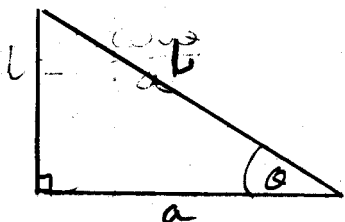
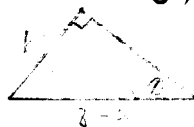


\therefore all integer values are greater than or equal to zero.

10) a) i) Similar triangles.



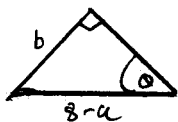
$$\sin \theta = \frac{6}{10} = \frac{3}{5} \quad \cos \theta = \frac{8}{10} = \frac{4}{5} \quad \tan \theta = \frac{6}{8} = \frac{3}{4}$$



$$\cos \theta = \frac{a}{l} \quad \therefore \frac{4}{5} = \frac{a}{l} = \frac{a}{\frac{4}{5}a}$$

$$\therefore \underline{\underline{l = \frac{5a}{4}}}$$

ii)



$$\sin \theta = \frac{b}{8-a} \quad \therefore b = \frac{3}{5}(8-a)$$

$$\therefore A = \frac{5}{4}a \times \frac{3}{5}(8-a) = \underline{\underline{\frac{3}{4}a(8-a)}}$$

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10) b) Max at $\frac{dA}{da} = 0$

$\therefore A = 6a - \frac{3}{4}a^2$

$\frac{dA}{da} = 6 - \frac{3}{2}a = 0$

$\therefore \frac{3}{2}a = 6$

$a = \frac{2 \times 6}{3} = \underline{\underline{4}}$

Check:

a	4^-	4	4^+
$\frac{dA}{da}$	$+$	0	$-$
shape	$/$	$-$	\backslash

\therefore Max at $a=4$